EFFECTIVE THERMAL CONDUCTIVITY OF A SATURATED POROUS MEDIUM IN THE PRESENCE OF PERCOLATION

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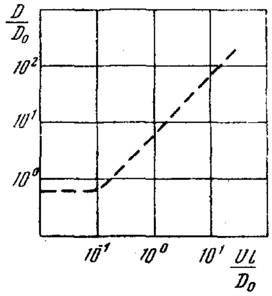
1. We shall consider a homogeneous percolation flow with percolation velocity u directed along the x axis. The time function of the temperature field T(x, t) (which for simplicity we shall also assume to depend only on x and t) will be governed by two processes—molecular heat conduction in the liquid and in the solid skeleton and convective heat transfer together with the liquid. It is characteristic of convective heat transfer in a porous medium that besides the ordered general motion of the liquid along the x axis there is a random time constant velocity field, a sort of "turbulence," that produces effective mixing of the liquid and consequently considerably accelerates all the transfer processes.



If we write the heat balance equation neglecting the spread of velocity, we get

$$C\frac{\partial T}{\partial t} + uC_0\frac{\partial T}{\partial x} = -\frac{\partial q}{\partial x}, \qquad (1)$$

where C is the heat capacity of unit volume of the porous medium saturated with liquid, C_0 is the heat capacity of unit volume of the liquid, and q is the heat flux. In the absence of motion the heat flux is governed by the thermal conductivity

$$q = -\lambda^*\frac{\partial T}{\partial x}, \qquad (2)$$

where λ^* is the thermal conductivity of the porous medium saturated with liquid at rest.

If we retain Eqs. (1) and (2) in the above form, then in the presence of percolation the effective thermal conductivity λ is a function of the percolation velocity u. The search for methods of applying heat to oil-bearing strata requires a correct estimation of the role played by heat conduction (flux q) as compared with convective transfer with the general liquid flow, for which it is necessary to know the dependence of λ on u. Over a broad range of velocities λ increases linearly with increase in u [1, 2]

$$\lambda = \lambda^* + Au. \qquad (3)$$

However, there are no reliable data on the value of the coefficient A in this velocity range, which is typical of oil industry practice (of the order of 10^{-2} cm/sec).

On the other hand, the process of diffusion of a neutral admixture ("tagged particles") in a porous medium has been quite well investigated (see, for example, [2, 3]). The effect of a percolation flow on diffusion (in the velocity range of practical interest) is extremely strong. The "effective diffusion coefficient", determined in the same way as the effective thermal conductivity, is several orders greater than the molecular diffusion coefficient. The analogy between the processes of heat conduction and diffusion suggests the thought that the effective thermal conductivity is similarly strongly dependent on the percolation velocity. Below it is shown that this is not so.

2. The thermal conductivity of a saturated porous medium may depend on the dimensional parameters u, l, C_0, C_1, λ_0, λ_1, ρ and on the dimensionless geometrical characteristics of the medium. Here λ_0 and λ_1 are the thermal conductivities of the liquid and the porous skeleton, C_0 is the volume heat capacity of the liquid, C_1 is the volume heat capacity of the skeleton, l is the internal scale of the porous medium (e.g., the mean pore radius). In our case we shall consider heat transfer processes and disregard the heat resulting from dissipation of energy in the motion of the liquid. Therefore the characteristic parameters do not include the viscosity μ. For the same reason we can take the dimension of thermal energy [Q] = cal as the independent dimension, without introducing new dimensional constants (mechanical equivalent of heat J) (see, for example, [4]). From the above seven quantities and the thermal conductivity λ it is possible to construct four dimensionless parameters

$$\frac{\lambda}{\lambda_0}, \quad \frac{ulC_0}{\lambda_0}, \quad \frac{\lambda_1}{\lambda_0}, \quad \frac{C_1}{C_0}.$$

These parameters do not contain the density of the liquid, the only quantity whose dimension includes mass. The density of the skeleton was never included in the characteristic parameters, since the heat conductivity inside the solid grains is not connected with the motion of the microparticles but proceeds at the molecular level. Hence

$$\lambda = \lambda_0 f\,(ulC_0 / \lambda_0,\ \lambda_1 / \lambda_0,\ C_1 / C_0). \qquad (4)$$

The linear relation (3), experimentally observed even at high percolation velocities in a coarsely granular medium [1], shows that it is possible to confine oneself to the first term of the expansion of the function f in powers of the first argument, so that we have

$$\lambda = \lambda^* + f_1'\,(0, \lambda_1 / \lambda_0, C_1 / C_0)\, ulC_0. \qquad (5)$$

Here the first term is equal to the heat conductivity of a porous medium saturated with liquid at rest, while the second takes into account the effect of motion. Obviously, motion of the liquid increases only the heat transfer in the liquid phase, leaving the heat transfer in the solid phase unchanged or reducing it as a result of equalization of the temperature gradients. Therefore the relative increase in the heat flux due to motion will be the greater, the smaller the intrinsic heat conductivity of the skeleton, so that

$$\frac{\lambda}{\lambda^*\,(0, \lambda_1/\lambda_0, C_1/C_0)} \leqslant 1 + \frac{f_1'\,(0, 0, C_1/C_0)\, ulC_0}{\lambda^*\,(0, 0, C_1/C_0)}.$$

At the same time, it is clear that if the heat conductivity of the skeleton is zero, its heat capacity cannot affect the heat transfer process in the percolation flow, so that

$$\lambda / \lambda^*\,(0, \lambda_1 / \lambda_0, C_1 / C_0) \leqslant 1 + f_1'\,(0, 0, 0)\, ulC_0 / \lambda^*\,(0, 0, 0). \qquad (6)$$

Relation (6) shows that the greatest relative increase in heat conductivity occurs when the skeleton does not participate in heat transfer.

We now note that the process of heat conduction in a porous medium with a thermally nonconducting skeleton is perfectly analogous (if the temperature gradients are small, so that it is possible to neglect the variation of the properties of the medium) to the process of propagation of a neutral admixture. Therefore, to estimate the derivative $f_1'(0, 0, 0)$ it is possible to use experimental data relating to diffusion in a porous medium. The figure, taken from [3], shows the ratio of the effective diffusion coefficient D to the molecular diffusion coefficient D_0 as a function of the parameter ul/D_0.

Obviously, the same graph will represent the ratio λ/λ_0 as a function of the parameter ulC_0/λ_0 for $\lambda_1 = 0$.

From the figure it is clear that over a broad range of percolation rates $f_1'(0, 0, 0) \sim 10$. Hence it follows that

$$\lambda \leqslant \lambda^* + aulC_0 \qquad (a = f_1'(0, 0, 0)\, \lambda^* / \lambda^*(0, 0, 0)). \qquad (7)$$

Here a is a quantity of the order of several tens. At $\lambda^* \sim 10^{-3}$ cal/cm · sec · deg, $C_0 \sim 1$ cal/cm^3 · deg, $l \sim 10^{-2}$ cm, $u_0 \sim 10^{-2}$ cm/sec values typical of a saturated porous medium both terms in (7) are of the same order. This shows that percolation flows typical of oil industry practice do not lead to a change in the order of magnitude of the thermal conductivity and for such flows one is still justified in neglecting the thermal conductivity in the direction of motion as compared with the convective transfer corresponding to the average motion.

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